A

Exam Seat No. ________

The Maharaja Sayajirao University of Baroda

Faculty of Science

ADMISSION ENTRANCE EXAMINATION-2018

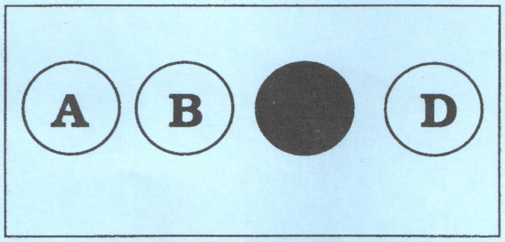
SUBJECT : MATHEMATICS

DAY & Date : Monday, 18th June, 2018 **TIME :** 10.00 a.m. to 11.00 a.m.

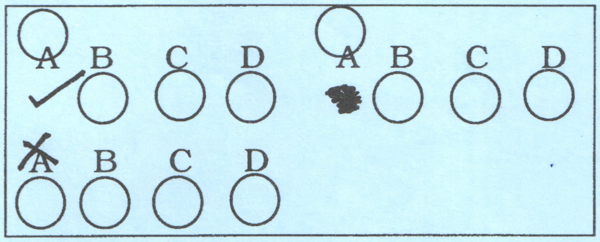
Important Instructions:

1. This test booklet is to be opened only when instructed by the invigilators to do so.
2. This booklet carries 50 questions on 6 printed pages. All Questions carry equal marks.
3. For every correct answer, candidate will earn 2 marks, 0.5 marks will be deducted for every wrong answer.
4. **Exam Seat Number** must be entered correctly in the OMR sheet, as advised by the invigilators. **The Question Booklet code (A, B, C or D)** must also be mentioned on the OMR sheet (if not printed already) as instructed.
5. Answers must be marked in the OMR sheet using **a black or dark blue ball point pen only**. The circle should be filled in completely, leaving no gaps.
6. **Gadgets** (Mobile phones, pagers, ear phones, music players, calculators etc.) **are strictly prohibited** in the exam hall. If any candidate is found in possession of any of these at his/her exam seat, he/she is liable to be disqualified.

Correct way of marking answer:



Incorrect way of marking answer:



Invigilator's Signature: ________

A1

1. For $n \in \mathbb{N}$, let $\mathcal{P}_n$ denote the real vector space of polynomials of degree less than or equal to n in one variable x. Define $T: \mathcal{P}_2 \to \mathcal{P}_4$ by $T\big(p(x)\big) = p(x^2)$. Then
   (A) T is a linear transformation and rank of T is 5
   (B) T is a linear transformation and rank of T is 3
   (C) T is a linear transformation and rank of T is 2
   (D) $T$is not a linear transformation .

2.Consider the matrix$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, where $\theta = \frac{2\pi}{31}$. Then A^{2015} equals

(A) A(B)I (C)$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (D)$\begin{bmatrix} \cos 31\theta & \sin 31\theta \\ -\sin 31\theta & \cos 31\theta \end{bmatrix}$.

3. The dimension of the real vector space $\mathbb{C}^3$ is

   (A) 1 (B) infinite (C) 3 (D) 6 .

4. Let U and W be finite dimensional subspaces of a vector space$V$with $\dim U = 5$, $\dim W = 4$and$\dim(U + W) = 7$. Then
   (A) $U \cup W$need not be a finite dimensional subspace of V
   (B) there exists a non-zero vector x such that $[U \cap W] = [x]$
   (C) there exists independent vectors $x_1,\ x_2 \in U \cap W$such that $[U \cap W] = [x_1,\ x_2]$
   (D) dimension of $U \cap W$ must be between 3 and 5 .

5. If the nullspace of a 2×5real matrix A is the line through the vector$u = (1, 1, 0,\ 0,\ 0)$, then the rank of the matrix$A$is equal to
   (A) 1 (B) 2 (C) 3 (D) 4 .

6. Which of the following statements is not true?
   (A) If V is a vector space having dimension n, then Vhas exactly one subspace with dimension 0 and exactly one subspace with dimension n
   (B) The dimension of real vector space $M_{m\times n}(\mathbb{R})$ is $m + n$
   (C) A vector space cannot have more than one basis
   (D) Every subspace of a finite dimensional vector space is finite dimensional .

7. Let $T: U \to V$ be a linear map and let $\{x_1,\ x_2,\ \dots,\ x_n\}$be a basis for$U$. Then the set $\{T(x_1),\ T(x_2),\ \dots,\ T(x_n)\}$is a linearly independent set in Vif and only if
   (A) $T$is one-one and onto (B) T is one-one
   (C) $T$is onto (D) $U = V$.

8.$\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(\sum_{k=1}^{n} \frac{1}{\sqrt{k}+\sqrt{k+1}}\right) =$

(A) 0 (B)1(C)-1 (D)∞.

A2

9. Let $\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$, and $\sum_{n=1}^{\infty} c_n$ be given three series of positive terms. The series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} c_n$ are convergent. If $\lim_{n\to\infty} \frac{a_n+b_n}{c_n} = 1$, then which one of the following statements is TRUE?

(A) $\sum_{n=1}^{\infty} b_n$ is divergent (B) nothing can be said about convergence of $\sum_{n=1}^{\infty} b_n$

(C) $\sum_{n=1}^{\infty} b_n$ is convergent (D) $\lim_{n\to\infty} b_n = 1$.

10. If $\sum a_n$ is a convergent infinite series of positive terms , then which of the following holds always?

(A) $\sum \frac{\sqrt{a_n}}{n}$ is convergent (B) $\sum n\sqrt{a_n}$ is convergent

(C) $\sum \sqrt{a_n}$ is convergent (D) $\sum \frac{1}{\sqrt{a_n}}$ is convergent .

11. Which one of the following series is convergent ?

(A) $\sum_{n=2}^{\infty} \frac{1}{\log n}$ (B) $\sum_{n=2}^{\infty} \frac{1}{n\log n}$ (C) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\log n}}$ (D) $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^3}$.

12. $\sum_{n=3}^{\infty} \frac{1}{n!} =$ ________.

(A) $\frac{2e-5}{2}$ (B) e (C) $e-1$ (D) $e-2$.

13. Which one of the following series is absolutely convergent?

(A) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ (B) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2+3n+2}$ (C) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{2n-1}}{n}$ (D) $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\sqrt{n}}$.

14. The general solution of the equation $y'' - f(x)y' + [f(x)-1]y = 0$ is

(A) $y = c_1e^x + c_2e^{f(x)}$ (B) $y = c_1e^x + c_2e^{\int f(x)dx}$
(C) $y = c_1e^x + c_2e^x \int e^{[-2x+\int f(x)dx]}\,dx$ (D) $y = c_1e^{-x} + c_2e^{-x+f(x)}$.

15. If ϕ_1 and ϕ_2 are two linearly independent solutions of the differential equation $y'' + a_1(x)y' + a_2(x)y = 0$, where $a_1(x)$ and $a_2(x)$ are continuous functions defined on $[a,b]$ and suppose that $\phi_2(x) \neq 0$ for all x in $[a,b]$. If $W(\phi_1,\phi_2)(x)$ denotes the value of the Wronskian at x , then

(A) $W(\phi_1,\phi_2)(x) = 0 \forall x \in [a,b]$ (B) $W(\phi_1,\phi_2)(x) = a_1(x)a_2(x)$

(C) $\left(\frac{\phi_1(x)}{\phi_2(x)}\right)' = -\frac{W(\phi_1,\phi_2)(x)}{[\phi_2(x)]^2}$ (D) $W(\phi_1,\phi_2)(x) = constant\ \forall x \in [a,b]$.

16. The differential equation whose linearly independent solutions are x and x^2-1 is

(A) $y'' - 2x(x^2+1)y' + 2y = 0$ (B) $(x^2+1)y'' - 2xy' + 2y = 0$
(C) $(x^2-1)y'' - 2xy' + 2y = 0$ (D) $xy'' + x^2y' + 2y = 0$.

A3

17. The integral curves of the set of equations $\frac{dx}{z-y} = \frac{dy}{x-z} = \frac{dz}{y-x}$ are
   (A) two parameter family of ellipses
   (B) two parameter family of parabolas
   (C) two parameter family of straight lines
   (D) two parameter family of circles .

18. The complete integral of the equation $z = px + qy + \sqrt{p^2 + q^2 + 1}$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$, represents
   (A) all planes at unit distance from origin (B) all planes passing through origin
   (C) all spheres with centre origin (D) all ellipses with centre as origin .

19. First order divided difference of the function $f(x) = 1/x^2$ at $x = a, b$, is
   (A) $\frac{a+b}{a^2\, b^2}$ (B) $\frac{a+b}{ab}$ (C) $-\frac{a+b}{a^2\, b^2}$ (D) $-\frac{a+b}{ab}$.

20. The error occurring in approximating the function $f(x)$ in (a, b) using Lagrange's interpolating polynomial $P_1(x)$ for $x = x_0,\ x_1$ is
   (A) $f'(\xi)(x - x_0)(x - x_1)$ (B) $\frac{1}{2}f''(\xi)(x - x_0)(x - x_1)$
   (C) $\frac{1}{6}f'''(\xi)(x - x_0)(x - x_1)$ (D) $\frac{1}{3}f'''(\xi)(x - x_0)(x - x_1)$.

21. The general iteration method $x = g(x)$ converges in (a, b) if
   (A) $|g(x)| < 1$ (B) $|g'(x)| < 1$
   (C) $g(a) < g(x) < g(b)$ (D) $|g'(x)| > 1$.

22. In usual notations of Numerical analysis operators, $\nabla + \mathrm{E}^{-1} =$
   (A) Δ (B) E^{-1} (C) ∇ (D) I.

23. Which of the following is the equation of circle with center at $(5,0)$ and radius 5 in polar coordinates
   (A) $r = 5$ (B) $r = 10\ cos\theta$ (C) $r = 10\ sin\theta$ (D) $r = 5\ sin\theta$.

24. The equation of cone with vertex at origin and which passes through the circle $x^2 + y^2 = 1,\ z = 2$ is
   (A) $4x^2 + 4y^2 = z^2$ (B) $x^2 + 4y^2 = z^2$
   (C) $4x^2 + y^2 = z^2$ (D) $x^2 + y^2 = z^2$.

25. Equation of the sphere having the points $A(1,0,1)$ and $B(2,1,0)$ as diameter ends
   (A) $x^2 + y^2 + z^2 - 3x - y - z - 2 = 0$ (B) $x^2 + y^2 + z^2 - 3x - y - z + 2 = 0$
   (C) $x^2 + y^2 + z^2 - x - y - z - 2 = 0$ (D) $x^2 + y^2 + z^2 - x - y - z + 2 = 0$.

A4

26. Let : $\mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = \begin{cases} \frac{xy}{|x|} if x \neq 0 \\ 0 \quad elsewhere \end{cases}$. Then at the point (0,0), which of thefollowing statements is true ?

(A) $f$is not continuous (B) f is continuous
(C) $f$is differentiable (D) partial derivatives do not exist.

27. The value of the double integral $\int_0^{\pi} \int_0^{x} \frac{siny}{\pi - y} dydx$ is
(A) 0 (B) π (C) -2 (D) 2 .

28. If the triple integral over the region bounded by the planes $2x + 2y + z = 4, x = 0,$ $y = 0$ and $z = 0$is given by$\int_0^2 \int_0^{g(x)} \int_0^{f(x,y)} dzdydx$ then the function $2g(x) - f(x,y)$is

(A) $2y$(B) $2 - x - 2y$(C)$x + 2y - 2$ (D) $2x$.

29. In C++ programming language, the operator ?: is a/an

(A) Conditional operator (B) Relational operator
(C) Logical operator (D) Arithmetic operator.

30. Let $G = \left\{A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R} \& ad - bc \neq 0\right\}$. Then G is a group under matrix multiplication. Suppose H is a subgroup of G given by $H = \{A \in G \ : \ ad - bc = 1\}$. Then

(A) G/H is isomorphic to group $(\mathbb{R}, +)$
(B) G/H is isomorphic to group $(\mathbb{R}^* = \mathbb{R}\backslash\{0\}, \ \cdot)$
(C) $G/H$is isomorphic to group $(\mathbb{Q}, +)$.
(D) $G/H$is isomorphic to group $(\mathbb{Q}^* = \mathbb{Q}\backslash\{0\}, \ \cdot)$.

31. A group G is said to be a simple group if it has no proper normal subgroup. Let G be a simple abelian group with $o(G) = n$. Then

(A) $G$is a cyclic group and n is an even integer greater than 3.
(B) $G$is an abelian but not a cyclic group and n is an even integer greater than 3.
(C) $G$is an abelian but not a cyclic group and n is a prime integer.
(D) $G$is a cyclic group and n is a prime integer.

32. Let G be a group containing more than 12 elements of order 13. Then G is

(A) an infinite non – cyclic group (B) a finite non – cyclic group.
(C) a finite cyclic group. (D) an infinite cyclic group.

A5

33. Let p be a prime and let $\mathbb{Z}_p$ denote the set of all integers modulo p equivalence classes. Considerthe ring$(\mathbb{Z}_p, +_p, \cdot_p)$. Then which of the following statements is not true?

(A) $\mathbb{Z}_p$has no proper nontrivial ideal (B) $\mathbb{Z}_p$ has a proper nontrivial ideal
(C) $\mathbb{Z}_p$is a field (D) $\mathbb{Z}_p$ is a division ring.

34. Let G be a group with $o(G) = 3289$.

(A) Then G is cyclic and all p −Sylow subgroup of G are not normal
(B) Then G is non – cyclic and all p −Sylow subgroups of G are not normal.
(C) Then G is non – cyclic and all p −Sylow subgroups of G are normal
(D) Then G is cyclic and all p −Sylow subgroups of G are normal

35. Let $C([0,1])$ denote the set of all continuous real valued functions defined on $[0,1]$.
(A) $C([0,1])$ is a field
(B) $C([0,1])$ is not an integral domain
(C) $C([0,1])$ is an integral domain but not a field
(D) $C([0,1])$is a division ring.

36. If $f: [0,2\pi) \to S^1$, defined as $f(t) = (cos(t), sin(t))$, where S^1 is the unit circle with center (0,0) and radius 1 in XY-plane, then

(A) its inverse i.e.f^{-1} does not exists (B) f is continuous but f^{-1} is not continuous

(C) $f$andf^{-1} both are continuous (D) f^{-1} is continuous but f is not continuous .

37.Let E be a bounded infinite subset of $\mathbb{R}$. If $\alpha = sup\,\{x: x \in E\}$ then α is
(A) an interior point of E (B) always a member of E
(C) a limit point of E (D) neither an interior nor a limit point of E.

38. If $E_n = (-1 - \frac{1}{n}, 1 - \frac{1}{n})$, $n \in \mathbb{N}$, and E=$\bigcap_{n=1}^{\infty} E_n$, then $E =$

(A) $(-1, 1)$ (B) $[-1, 1]$ (C) $(-2, 0]$ (D) $[-1, 0)$.

39. With respect to the usual metric over the real line, every point of the set $E = \{0,1,2,\ldots\}$ is

(A) a limit point of E(B) an interior point of E
(C) an isolated point of E(D) a boundary point of E .

40. In the XY-plane, the set $E = \{(x, x): x \in \mathbb{R}\}$ is

(A) open (B) closed (C) neither open nor closed (D) open and closed .

41. If P is a set of all interior points and Q is a set of all limit points, of a non-empty subset E of a metric space (X, d) then

(A) $P = Q$ (B) $P \subseteq Q$ (C) $Q \subseteq P$ (D) P and Q may be disjoint .

A6

42. For given fixed natural numbers $k$and m; $\lim_{n\to\infty}\left[\frac{(n+1)^m+(n+2)^m+\cdots+(n+k)^m}{n^{m-1}}-kn\right]=$

(A) $\frac{km(k+1)}{2}$ (B)∞ (C) km (D) 0.

43. If $f:\mathbb{R}\to\mathbb{R}$ defined as $(x)=\begin{cases}x, & if x is rational\\ 0, & if x is irrational\end{cases}$,then f is

(A) everywhere discontinuous (B) continuous at rational only
(C)continuous at irrational only (D) continuous at exactly one point only.

44. Suppose that a and c are real numbers, c>0 and f is defined on [0,1] as
$f(x)=\begin{cases}x^a\sin(x^{-c}), & if x>0\\ 0, & if x=0\end{cases}$. Then $f'$is bounded if and only if

(A) $a>0$ (B) $a>1$ (C) $a\geq 1+c$ (D)$a>1+c$.

45. If a real function f is defined over $\mathbb{R}$ such that $|f(x)-f(y)|\leq(x-y)^2,\forall x,y\in\mathbb{R}$, then
(A) f is strictly monotonic (B)f is constant
(C) f'' need not exists (D)f'' exists but need not be continuous .

46. If $p>1,\ u\geq 0, v\geq 0$ and q is such that $\frac{1}{p}+\frac{1}{q}=1$, then $uv\leq\frac{u^p}{p}+\frac{v^q}{q}$. Here, the equality holds if and only if

(A) u^p=v^q (B)u^q=v^p (C) $u=v$ (D)$p=q$.

47. The series $\sum_{n=1}^{\infty}(-1)^n\left(\frac{x^2+n}{n^2}\right)$ of real numbers
(A) diverges for any real x
(B) converges uniformly on every bounded interval but does not converges absolutely for any real$x$
(C) converges uniformly in $\mathbb{R}$
(D) converges absolutely for any real x .

48 . If $S=\{x\in R : x\geq 0 , 2|\sqrt{x}-3|+\sqrt{x}(\sqrt{x}-6)+6=0\}$, then S
(A) is an empty set (B) contains exactly one element
(C)contains exactly two elements (D)contains exactly four elements .

49. If $a,b,x,y\in R$such that$a-b=1 y\neq 0$ and the complex number$z=x+iy$ satisfies $Im\left(\frac{az+b}{z+1}\right)=y$, then x is equal to:
(A)$-1\pm\sqrt{1-y^2}$ (B)$\pm 1\pm\sqrt{1-y^2}$
(C) $-1\pm\sqrt{1+y^2}$(D)$\pm 1\pm\sqrt{1+y^2}$.

50. If$1,\alpha_1,\alpha_2,\dots,\alpha_{99}$ are the distinct 100^{th} roots of unity then $\sum_{1\leq i<j\leq 99}(\alpha_i\alpha_j)$ is equal to :
(A) 100 (B) 99 (C) 1 (D) 0 .

______THE END______

INSTRUCTIONS FOR ENTRANCE TEST

1. There are 50 (fifty) questions in this question paper.
2. No candidate will be allowed to leave the examination hall till the examination is over.
3. **USE ONLY BLUE/ BLACK BALL POINT PEN FOR WRITING THE ANSWER IN OMR SHEET.**
4. Candidate must write Exam Seat Number on OMR sheet from extreme right.
5. Answer all the questions by putting an appropriate choice, out of the alternatives A, B, C and D against each question number on the OMR sheet.
6. All questions are compulsory.
7. There is a **NEGATIVE MARKING**. Each question correctly answered carries **2 marks**, each question incorrectly answered carries **–0.5 marks**and each question not attempted carries **0 marks.**
8. Rough work, if required, may be done on the blank sheet attached at the end.
9. Use of calculator is not permitted.
10. Cell phone is not permitted in the examination hall.
11. Any candidate found copying will be immediately removed from the Examination hall and his/ her admission to M. Sc. Mathematics will not be considered.

GENERAL INSTRUCTION FOR ADMISSION

1. Candidate is required to attach self – attested copy of the following documents with his/ her copy of the downloaded application form:
   - B. Sc. Semester – 6 mark sheet (if in Semester System)
   - F. Y. B. Sc., S. Y. B. Sc. and T. Y. B. Sc. mark sheets (if in annual system)
   - Certificate for SC/ ST/ SEBC/ EBC candidates
   - Non – creamy layer certificate (if SEBC candidate)of current year
   - Eligibility certificate for students with B. Sc. other than The M. S. University students.
2. In case original mark sheet of last examination is not issued by the University, the print out of mark sheet on website may be attached.
3. BA Mathematics students of The M. S. University of Baroda are required to attach transfer certificate.