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ENTRANCE TEST SEAT NO.___________

DEPARTMENT OF MATHEMATICS
FACULTY OF SCIENCE
THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA
VADODARA – 390 002

Entrance Test for Admissions to

M. Sc. Mathematics 2016 – 2018

Day &Date : Monday, June 20, 2016
Time : 10.00 am to 11.00 am

Total Marks : 100

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Signature of Junior Supervisor and Date

1. $\left(\dfrac{1+\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}}{1+\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}}\right)^{36}=$ ________ .

(A) -1 (B) 1 (C) 0 (D) ½

2. $(-i)^{(-i)^{(-i)}}=$ _______ .

(A) $\dfrac{i\pi^2}{4}$ (B) $-i$ (C) i (D) $\dfrac{-i\pi}{2}$

3. If $x_n=\cos\dfrac{\pi}{2^n}+i\sin\dfrac{\pi}{2^n}$ then $\prod_{n=1}^{\infty} x_n=$ ________ .

(A) $-i$ (B) -1 (C) i (D) 1

4. If $a=\cos\alpha+i\sin\alpha$, $b=\cos\beta+i\sin\beta$, $c=\cos\gamma+i\sin\gamma$ *and* $\dfrac{a}{b}+\dfrac{b}{c}+\dfrac{c}{a}=1$, then $\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)=$ _________ .

(A) 3/2 (B) -3/2 (C) 0 (D) 1

5. A cone passing through the coordinate axes and having origin as its vertex has the equation
(A) $cz^2+2fyz+2gzx=0$ (B) $ax^2+2gxz+2hxy=0$
(C) $fyz+gzx+hxy=0$ (D) $by^2+2fyz+2hxy=0$

6. The right circular cylinder whose axis is z-axis and radius r, has the equation:
(A) $x^2+y^2=r^2$ (B) $y^2+z^2=r^2$
(C) $x^2+z^2=r^2$ (D) $x^2+y^2+z^2=r^2$

7. The equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ represents
(A) Ellipsoid (B) Hyperboloid of one sheet
(C) Hyperboloid of two sheet (D) Sphere

8. Which of the following represents the equation of straight line in space?
(A) $ax+by+cz+d=0;\quad px+qy+rz+s=0$
(B) $x^2+y^2+z^2=r^2;\quad ax+by+cz+d=0$
(C) $x^2+y^2+z^2=r^2;\quad x^2+y^2+z^2=s^2\quad (s\neq r)$
(D) $ax+by+cz+d=0;\quad x^2+z^2=r^2$

9. If Matrix $A=\begin{pmatrix}4 & x+2\\ 2x-3 & x+1\end{pmatrix}$ is symmetric then x is equal to

(A)2 (B) 3 (C)4 (D)5

10. The Matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix}$ is nilpotent of index (or order) :

(A)1 (B)2 (C)3 (D)4

11. The Matrix $\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1+i \\ 1-i & -1 \end{pmatrix}$ is

(A)Unitary (B)Idempotent (C)Orthogonal (D)None of these

12. Let W be a subspace of a vector space V and let S be a linearly dependent set in W. Which of the following statement is false?

(A) If S_1 is a subset of V such that $S \subseteq S_1$ then S_1 must be linearly dependent.

(B) If $x \in V$ and $x \notin W$ then $S \cup \{x\}$ is linearly dependent.

(C) There exists a subset $S_1 \subset S$ such that S_1 is linearly independent.

(D) Every nonempty subset of S is linearly dependent.

13. Let U and W be subspaces of a vector space V. Then which of the following subset of V need not be a subspace of V.

(A) $U+W$ (B) $U \cup W$ (C) $U \cap W$ (D) $U \oplus W$

14. The dimension of the real vector space C^2 is

(A) 1 (B) 2 (C) 3 (D) 4

15. Let $T : R^5 \to R^2$ be a linear transformation. Then $rank(T) + nullity(T)$ is equal to

(A) 3 (B) 2 (C) 5 (D) 7

16. The number of vector subspaces of the real vector space R is

(A) 0 (B) 1 (C) 2 (D) infinite

17. Let X_n denote the group of n^{th} roots of unity and let $G = \bigcup_{n=1}^{\infty} X_n$. Then

(A) $G$is an abelian non – cyclic group in which order of every element is finite.

(B) $G$is a cyclic group in which order of every element is finite.

(C) $G$is a non abelian group in which order of every element is finite.

(D) $sG$is an abelian group which has at least one element of infinite order.

18. For $n \geq 3$, consider the groupS_n of permutations of$n$ – elements. If$Z(S_n)$ denotes the center of S_n, then

(A) $Z(S_n) = \{Id\}$, where Id is the identity permutation. (B) $Z(S_n) = A_n$

(C) $Z(S_n) = S_n$ (D) None of the above

19. Let G be a finite abelian simple group with order of Gas $o(G)$. Then

(A) $o(G) = p$, where p is a prime integer.

(B) $o(G) = p^2$, where p is a prime integer.

(C) $o(G) = 2^n$, where $n \geq 3$.

(D) No information about the order of G is known.

20. Let G be a cyclic group of order 100. Then the number of subgroups of G are

(A) 9 (B) 40 (C) 10 (D) 50

21. Let G be a group of order 35. Then

(A) $G$is a cyclic group.

(B) $G$is an abelian non – cyclic group.

(C) $G$is not an abelian group.

(D) No information about G is known.

22. Which of the following statements is True ?

(A) The sequence $\left\{\frac{1}{n}\right\}$ and series $\sum_{n=1}^{\infty}\frac{1}{n}$ both converge.

(B) The sequence $\left\{\frac{1}{n}\right\}$converges and the series $\sum_{n=1}^{\infty}\frac{1}{n}$diverges.

(C) The series $\sum_{n=1}^{\infty}\frac{1}{n}$ converges and the sequence $\left\{\frac{1}{n}\right\}$diverges.

(D) The sequence $\left\{\frac{1}{n}\right\}$ and series $\sum_{n=1}^{\infty}\frac{1}{n}$ both do not converge.

23. If$\lim_{n\to\infty} x_n = \ell$ then $\lim_{n\to\infty}\frac{x_1+x_2+\ldots+x_n}{n} =$

(A) 1 (B) 0 (C) ℓ (D) $1/\ell$

24. Sum of the convergent series 0.612612612… is

(A) $\frac{68}{111}$ (B) $\frac{111}{68}$ (C) 68 (D) 111

25. The infinite series whose n^{th} partial sum $S_n = \frac{n}{2n+1}$ is given by

(A) $\sum_{k=1}^{\infty}\frac{1}{4k-1}$ (B) $\sum_{k=1}^{\infty}\frac{1}{(4k-1)^2}$ (C) $\sum_{k=1}^{\infty}\frac{1}{4k^2-1}$ (D) $\sum_{k=1}^{\infty}\frac{1}{16k-1}$

26. The sum of the series $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}$ is

(A) not defined (B) sin 1 (C) 0 (D) cos 1

27. Let f and g be two continuous functions from $\mathbb{R}$ into $\mathbb{R}$. The set $\{x \in \mathbb{R} : f(x)=g(x)\}$ is always

(A) compact (B) open (C) closed (D) finite

28. Let d and d* be the usual and discrete metric over $\mathbb{R}$, the set of real numbers, respectively. If $f:(\mathbb{R},d^*)\to(\mathbb{R},d)$ defined as $f(x)=[x],\ where\ [x]$ is the integral part of x, $\forall\, x$.Then$f$is

(A) discontinuous at only one point 0 (B) discontinuous at every integer

(C) discontinuous at every natural number (D) everywhere continuous

29. Let A be a closed set and B be its compliment in a connected metric space X, then

(A) diam $(A,B)>0$ (B) $A\cap\bar{B}=\emptyset$ (C) diam $(A,B)=0$ (D) diam $(A,B)<0$

30. With respect to the usual metric the set of integers $\mathbb{Z}$ is
(A)compact (B) compact and connected
(C) complete (D)compact but not connected

31. If f(x)=x^2, $\forall x \in \mathbb{R}$. Then f
(A) is bounded (B) has unique fixed point
(C) has no fixed point (D) is not uniformly continuous

32. The power series $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converges on
(A) $[-1,1)$ (B) $[-1,1]$ (C) $(-1,1)$ (D) $[-1,0)$

33. The series $\sum_{n=1}^{\infty} \frac{\cos nx}{n^3}$, $0 < x < 1, n = 1,2, ...,$ is
(A) divergent
(B) convergent but not uniformly convergent
(C)absolutely convergent but notuniformly convergent
(D)absolutely and uniformly convergent

34. If $f_n(x) = \frac{\sin nx}{\sqrt{n}}, x \in \mathbb{R}, n = 1,2, ...,$ then,
(A)$\{f_n\}$ is divergent
(B) $f_n \to f$, but f is not differentiable
(C)$f_n \to f, f$ is differentiable but $f_n' \nrightarrow f'$
(D) $f_n \to f, f$ is differentiableand $f_n' \to f'$

35. If P and Q are partitions of the interval $[a, b]$ and Q is a refinement of P, then for any bounded real function f and for any monotonically increasing function α defined on $[a, b]$,
(A) $U(Q, f, \alpha) \leq L(P, f, \alpha)$ (B) $L(Q, f, \alpha) \leq L(P, f, \alpha)$
(C) $L(P, f, \alpha) \leq L(Q, f, \alpha)$ (D) $U(P, f, \alpha) \leq U(Q, f, \alpha)$

36. A function f defined on $[a, b]$ need not be Riemann integrable on $[a, b]$, if
(A)$f$is increasing (B)f is bounded
(C)$f$is decreasing (D)f is continuous

37. If the differential equation $(x + k^2 siny)dx + [(2k - 1)x\, cosy - 2y]dy = 0$ is exact , then the value of k is
(A) 1 (B) -1 (C) ± 1 (D) 2

38. The Wronskian of the functions x^3 and $|x^3|$ on the interval $[-1,1]$ is
(A) 0 (B) $3x^2$ (C) $-3x^2$ (D) 1

39. If $y(x) = e^{6x}$ is a solution of the equation $\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 36y = 0$, then the other linearly independent solution of the equation is
(A) e^{-6x} (B) $6x^{6x}$(C) xe^{6x} (D)x^2e^{6x}

40. The integral curves of the system $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$ are

(A) parabolas (B) straight lines
(C) circles (D) hyperbolas

41. The complete integral of the non-linear PDE $p^2q^2z = p^3q^2x + p^2q^3y + p^2 + q^2$ is

(A) $z = ax + by + \frac{1}{a^2} + \frac{1}{b^2}$ (B) $z = ax + by$
(C) $z = ax^2 + by^2$ (D) $z = abxy$.

42. The function $f(x, y) = \dfrac{3xy}{x^2 + y^2}$ is not continuous at (0,0) because

(A) $f(0,0)$is not defined.
(B) $\lim_{(x,y)\to(0,0)} f(x, y)$does not exist
(C) $\lim_{(x,y)\to(0,0)} f(x, y)$exists but $f(0,0)$ is not defined
(D) both (A) and (B) are true

43. If f is a function of three variables, then the number of fourth order partial derivatives of f at a point, in general, is

(A)81 (B) 64 (C) 12 (D) 27

44. If f is a differentiable function of three variables and $H = f(y - z, z - x, x - y)$ then $H_x + H_y + H_z$ is

(A) 1 (B) 0 (C) -1 (D) not defined

45. On converting into polar coordinates, the double integral $\int_0^1 \int_0^x dydx$ becomes:

(A) $\int_0^{\pi/2} \int_0^1 rdrd\theta$ (B) $\int_0^{\pi/2} \int_0^{\sec\theta} rdrd\theta$ (C) $\int_0^{\pi/4} \int_0^{\sec\theta} drd\theta$ (D) $\int_0^{\pi/4} \int_0^{\sec\theta} rdrd\theta$

46. The total work done in moving an object around the square with vertices at (0,0), (0,2), (2,2) and (2,0) by the force field $F(x, y) = (x + 4y)\vec{\imath} + (y^2 + 5x)\vec{\jmath}$ is

(A) 2 (B) 1 (C) 0 (D) None of these

47. The number 0.0009875 when rounded off to three significant digits

(A) 0.001 (B) 0.000987 (C) 0.000988 (D) None of these

48. Order of convergence of Newton – Raphson method is

(A) 1 (B) 1.618 (C)2 (D) None of these

49. Which of the following is internal memory?

(A) Disks (B) Pen drives (C) RAM (D) CDs

50. Which unit holds data permanently?

(A) Input unit (B) secondary storage unit
(C) output unit (D) Primary memory unit